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Examination of the Relation between Science High School Students' Geometry Success and Van Hiele Geometric Thinking Levels

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ABSTRACT

The aim of this study is to detect the relation between Science High School students' success in geometry lessons and Van Hiele geometric thinking levels. The survey model, which is one of the quantitative research methods, was used in the study. The study group of the research consists of 244 students studying at three Science High Schools in the city center of Diyarbakır in the 2020-2021 academic year, selected by purposive sampling. The data were obtained by using the VHGTLT consisting of 25 questions suitable for the grade level of the students, and the GST consisting of 25 questions prepared by the researchers. The data were analyzed by calculating descriptive statistics, and the relation between the VHGTLT and the GST scores were analyzed with Pearson correlation. Moreover, according to both tests, one-factor analysis of variance technique was applied to detect whether there was a meaningful difference between Science High Schools. The results of the study are as follows: It was observed that most of the students (63.6%) who participated in the study were at or above Level 3 (Informal Inference) Van Hiele geometric thinking level. A middle correlation was found between the students' points obtained from both test results. In addition, as a result of the point obtained from both tests, a meaningful difference was found between the Science High School, which received the highest point in the province according to the high school entrance exam point, and the other two Science High Schools.

Keywords: Van Hiele level, Geometric thinking, Geometry success, Science high school students.

Introduction

Mathematics, which has its own systematic logic, is one of the indispensable elements of our daily life and is an important tool in learning other branches of science (Laurens, Batlolona, Batlolona, &Leasa, 2018). Mathematics and science are the basis of countries' development. For this reason, a large amount of time is allocated to mathematics education, which is the cornerstone of all sciences, in all stages of education, starting from the pre-school period (Alshatri, Wakil &Bakhtyar, 2019). Mathematics teaching has an important place in gaining knowledge and skills, understanding the social environment and developing creative thinking (Algani, 2019). One of the important sub-learning areas of mathematics education used in science, art and daily life is geometry(National Council of Mathematics Teachers [NCTM], 2000).

Geometry is a sub-learning field that deals with points, lines, planar and spatial geometric shapes, which are the figural part of mathematics, and the relation between these shapes, and the properties of these geometric shapes such as length, angle, and area (NCTM, 2000). By providing students with the opportunity to get to know their environment, geometry functions as a tool in their studies on science and other sciences related to mathematics (İlhan&Tutak, 2021). Thanks to geometry, students can analyze and solve problems, relate mathematics to real life and understand abstract concepts more easily (Dobbins, Gagnon, & Ulrich, 2014; Duatepe, 2000). Thus, in the studies conducted by the NCTM in the USA, the importance of geometry in mathematical proof and reasoning was emphasized (NCTM, 2000). These explanations show how important geometry teaching is for mathematics and daily life (van de Walle, 2013).

Geometry teaching creates the fun part of mathematics for students in the formation of mathematical concepts and information in the mind, starting with playful activities (Çiftçi& Tatar, 2014; Yi, Flores, & Wang; 2020). However, geometry teaching, which has a process from easy to difficult due to its structure, is seen as a lesson that is not liked and seen as difficult by most of the students despite its positive features (Çelebi-Akkaya, 2006). Similarly, Mistretta (2000) revealed in his study that students could not make strong conceptual meanings in geometry, which is a sub-learning area of mathematics. In an effective geometry teaching, it is important to plan students' thinking levels in geometry (Chang, Sung, & Lin, 2007; Regina, 2000).

It was suggested by Pierre Van Hiele that the development of geometry, whose beginning was built on the axiomatic system, in children's minds is in a hierarchical five-level structure, and he stated that children cannot reach the next level without assimilating one level (Wai, 2005). The development of these levels is given below.

1. Level 1 (Visual Period): In this period, students cannot comprehend shapes by adhering to geometric definitions. By observing their environment, they compare and name them by making use of examples from daily life (Pesen, 2008). At this level, shapes are recognized as a whole. Students say, "This is a rectangle because it looks like a door and a window." (Clements & Battista, 1990: 356; Battista & Clements, 1995). They can comment on geometric shapes by looking at their appearance. In this period, students perceive objects as they see them, but cannot notice the properties of objects (Hoffer, 1981).

2. Level 2 (Analysis Level): At this level, the class is considered, not the shape itself. Students don't think about just one rectangle; they think about all rectangles. They think that the opposite sides of the rectangle class are parallel and of equal length, have four sides, have four right angles, have equal diagonal lengths, and so on. They make groupings according to the characteristics of the shapes. They leave the shape and size of the figures in the background. If a shape is in the form of a cube, it must have all the features of the cube, that is, it must have six square surfaces equal to each other (Van de Walle, 2013). At this level, while describing shapes, students know all the features of that shape, but they do not know that the shapes are subclasses of each other, for example, that all squares are rectangles and all rectangles are parallelograms (Şahin,2008). The products of this level consist of knowing the properties of shapes (Van de Walle, 2013).

3. Level 3 (Informal Inference): This level is the level where students can see the relationships between geometric shapes. In this period, students can now make connections between geometric shapes and make sense of them. Although the logical implications are not yet understood, the definitions and axioms have become meaningful to students. For example, they can associate the properties of geometric shapes with each other, such as "Every square is a rectangle". On the other hand, students can observe the proof of the relationship, but they cannot (Hoffer, 1981). Although this level depends on the past education of the students, it generally corresponds to the students at the secondary school level (Olkun&Toluk, 2007: 225).

4. Level 4 (Formal Deduction/Inference): The important indicator that distinguishes this level from another level is that students can make geometrical proofs. They do these proofs with the help of theorems they have learned before (Olkun&Toluk, 2007). Students can be successful in the reasoning process by using the inductive method (Pesen, 2008). They can make inferences about geometrical properties related to abstract propositions. For example, they can prove from inferential propositions that the diagonals of the rectangle average each other. The products of this level are axiomatic systems based on inferences from geometry. The most important difference that distinguishes level 4 from level 3 is that the way of thinking is informal or formal (Van De Walle, 2013, p.404). At this level, students can think about the properties of shapes independently from the whole. This level corresponds to the high school years (Altun, 2008).

5. Level 5 (Most advanced period/Seeing the Relationships/Rigor): The individual who reaches this level can see the differences between different axiomatic systems and detect the relation between them. Can explain and apply the definitions, axioms and theorems of Euclidean geometry within non-Euclidean geometries (Hoffer, 1981). At this level students, can consider geometry like a discipline and conduct studies(Altun, 2008). This level corresponds to the undergraduate and graduate years (Pesen, 2008, p. 274).

These levels were expressed as 0-4 by Van Hiele Geldof (cited in Usiskin, 1982). Later, studies were conducted in which these levels were expressed as 1-5 (Hoffer, 1981; Senk, 1983; Aksu, 2005). The use of geometric thinking levels in the form of 1-5 allows the use of "0" level for individuals who cannot reach the visual level, which is the first step of the levels (Senk, 1983: 310).

The transition between levels is not dependent on age. The transition between levels depends on the quality of the education given (Duatepe-Paksu, 2016). Students at different educational levels may be at the same geometric thinking level. For this reason, it is necessary to plan and implement geometry teaching in accordance with the learning and development of students in order to ensure that geometry teaching achieves the desired goals. Pierre Van Hiele and Diana Van Hiele-Geldof saw that students had difficulties in learning geometry and developed a model suitable for students' learning and development levels, taking into account the places they had difficulties in geometry (Terzi, 2010).

In the literature review, it was observed that the geometric thinking levels were found to be low in general in the studies conducted to determine the geometric thinking levels of Van Hiele (Chang, Sung, & Lin, 2007; Hurma, 2011; Kutluca, 2013; Yi, Flores, & Wang, 2020). It has been observed that there is no study to determine the relation between the general success of high school geometry lessons and Van Hiele geometric thinking levels. Moreover, there is no study in the literature on geometric thinking levels for Science High School students, especially students with the highest point in the high school entrance examination. It is important to investigate whether there is a similar situation for Science High School students, who are the students with the highest point in the high school entrance examination, due to the characteristics of these high schools. Therefore, it is thought that there is a need for a study on the geometric thinking levels of Science High School students.

Based on these statements, the aim of this study is to detect the relation between Science High School students' success in geometry lesson and Van Hiele geometry thinking levels according to the results of geometry success test and Van Hiele geometry thinking test. For this purpose, the question, *"How is the relation between Science High School students' geometry lesson success and Van Hiele geometric thinking levels?"*, constitutes the main problem of the research. In this context, the sub-problems of the research are:

1. According to the results of the Van Hiele geometric thinking test, what is the distribution of the Van Hiele geometric thinking levels of the Science High School students?

2. Is there a meaningful relation between the point of Science High School students from the Van Hiele geometric thinking test and the GST?

3. Is there a meaningful difference among the science high schools point of results of the Van Hiele geometric thinking test (VHGTLT)?

4. Is there a meaningful difference among science high schools point of GST results?

5. Does the Van Hiele geometric thinking level of Science High School students show a meaningful difference compared to the GST?

Method

Model of the Research

This study, which aims to determine the relation between science high school students' geometry success and Van Hiele geometric thinking levels, was carried out with the survey model, which is one of the quantitative research designs. The approach that aims to describe a past or present situation as it is, without outside interference and influence, is called the survey model (Karasar, 2009, p. 77).

Participants

The schools to be studied in this study were selected by the purposeful sampling method. The reason for the purposeful selection of the schools is that the schools to be studied are in the city center of Diyarbakir in Turkey, consisting of students from the state science high school, which has the highest entrance point to high schools in the province. The entry base scores of these high schools in 2020 are as follows: X Science High School (480,189), Y Science High School (463,019) and Z Science High School (462,980).

The study group of the research consists of 60 students studying atX Science High School in Yenişehir district of Diyarbakır in Turkey, 101 students studying at Y Science High School in Bağlar district of Diyarbakır in Turkey and 83 students studying at Z Science High School in Sur district of Diyarbakır in Turkey. All of the students are in the 12th grade, a total of 244 students participated in the study.

Data Collection Tools

In the study, data were collected using the Van Hiele geometric thinking level test (VHGTLT) and geometry success test (GST).

Van Hiele Geometric Thinking Level Test

The VHGTLT is a level determination test developed by Usiskin (1982) to quantitatively determine the students' understanding of Van Hiele geometry. The Turkish translation and validity-reliability studies of this test were previously tested by Duatepe (2000).TheVHGTLT consists of 5 multiple-choice questions corresponding to each level and includes 25 questions in total. In addition, in the study of Usiskin (1982), the reliability coefficient at each level of the test varies between 0.65 and 0.79, and in the study of Duatepe (2000) it varies between 0.59 and 0.79. The distribution of the questions of the test according to the levels is given in Table 1.

Table 1. Distribution of Van Hiele Geometric Thinking Test Questions by Levels

Questions	Level of questions
1-5	Level -1
6-10	Level -2
11-15	Level -3
16-20	Level -4
21-25	Level -5

Each level of the VHGTLT has its own characteristics, and the transition between levels has a hierarchical structure. In this study, in order to ensure a hierarchical transition between levels, the correct answer of at least three questions out of five questions at each level was accepted as an indication that the student reached that level. In other words, the student who answers at least 3 of the first 5 questions correctly is at level 1 (visual term), if the student who reaches level 1 correctly answers at least 3 of the second 5 questions, he/she is at level 2 level (analysis). Even if the questions at the higher level are answered correctly enough without reaching any lower level, the higher level cannot be reached.

In the study, 25 questions of the VHGTLT were applied to detect the relation between Science High School students' success in geometry lessons and Van Hiele geometric thinking levels. The level of students who could not reach any level was accepted as zero.

Geometry Success Test

A 25-question multiple-choice test was prepared by the researcher in order to measure the geometry achievement of the students. While preparing the test, the achievements in the geometry learning field of the Ministry of National Education (MoNE) mathematics curriculum, which was put into practice in 2018, were taken into account. 8 Mathematics education experts opinions and 5 high school teachers' opinions were taken for the test. The questions in the test consist of geometry acquisitions belonging to the 9th, 10th and 11th grades. The questions were selected from the University Entrance Exam questions of the Student Selection and Placement Center (ÖSYM) from 2014 to 2019, the Achievement Comprehension Tests of the MoNE and the material questions of the Educational Information Network (EBA) of the MoNE for secondary education. In order to better determine the relation between Van Hiele geometric thinking levels and geometry success, trigonometry achievements and analytical geometry achievements, which are among the geometry achievements in the mathematics curriculum, were not included in the geometry success test. Experts and teachers gave a positive opinion that the questions in the test consisted of questions that could measure the geometry success of the 9th, 10th, and 11th grades in the 2018 Secondary School Mathematics Curriculum.
The GST developed by the researcher was applied as a pilot application to 67 students in the Anatolian High School of the Competition Authority, which received the highest score after the Science High Schools, which is the subject of the research. In order to make the item analysis of the GST, a ranking was made from the highest score to the lowest score of the test. The item difficulty level and item discrimination analysis levels of the questions belonging to the GST are given in Table 2 by taking the 27% upper quartile and 27% lower quartile scores in the score ranking.

Table 2. Item Analysis of the Questions in the Geometry Success Test

Item Number	Item Difficulty	Item Discrimination
Question 1	0.78	0.27
Question2	0.42	0.23
Question3	0.81	0.49
Question4	0.39	0.20
Question5	0.69	0.36
Question6	0.51	0.36
Question7	0.79	0.38
Question8	0.72	0.38
Question9	0.37	0.33
Question10	0.85	0.62
Question11	0.61	0.32
Question12	0.24	0.23
Question13	0.60	0.46
Question14	0.15	0.30
Question15	0.36	0.28
Question16	0.69	0.42

Question17	0.52	0.30
Question18	0.45	0.35
Question19	0.58	0.31
Question20	0.16	0.24
Question21	0.07	0.29
Question22	0.57	0.31
Question23	0.07	0.33
Question24	0.12	0.35
Question25	0.16	0.30
Average of the test	0.46	0.33

Item discrimination is the ability to distinguish items that are suitable or unsuitable for measuring the intended feature of the subject. Items with an item discrimination power of 0.40 and above have a very good discrimination ability, which indicates that the item is a quality item. Items with item discrimination power between 0.30 and 0.39 are quite good items, however, these items can be improved. Items with item discrimination power between 0.20 and 0.29 are medium-level items and these items should be developed and corrected. Items with item discrimination power between 0.10 and 0.19 are weak items and these items do not contribute to the test. If removing weak items from the test reduces the content validity of the test, weak items should be developed and corrected. Otherwise, these items should be discarded from the test. Items with negative item discrimination power are very bad items and should be removed from the test if they cannot be developed and corrected (Tekin, 2000). The item discrimination levels of the 1., 2., 4., 12., 15. and 20. questions in the geometry success test were between 0.20 and 0.29, and it was decided not to remove these items from the test after taking expert opinion. In addition, there is no item with an item discrimination level below 0.19 in the GST.

The reliability of the test was calculated according to the Cronbach Alpha coefficient and the reliability coefficient of the test was found to be 0.802. In the main application, the reliability coefficient of the test was found to be 0.778. The GST with 25 questions, of which validity and reliability analyzes were made, was applied to 244 students. Sample questions of GST are given in the appendix.

Data Analysis

The scoring system developed by Usiskin (1982) was used to determine the students' Van Hiele geometric thinking levels as points. In this scoring system, the weighted score to be obtained from each Van Hiele level is as follows (Usiskin, 1982).

0 points for students who do not answer 3 or more questions from any level 0 questions correctly.

1 point for students who correctly answer at least 3 of 5 questions

2 points for students who correctly answer at least 3 of the 5 questions for the answers to the questions between 6 and 10 of the 2nd level

4 points for students who correctly answer at least 3 of the 5 questions for the answers to the questions 11-15 of the 3rd level

8 points for students who correctly answer at least 3 of the 5 questions for the answers to the questions between 16 and 20 of the 4th level

For the answers to the questions between 21 and 25 belonging to the 5th level, 16 points were given to the students who answered at least 3 of the 5 questions correctly.

As a result, in this scoring system, students who do not answer 3 or more questions from any level correctly are assigned 0 points and assigned to 0 Level. Level 0 was also later termed the "semi-envisioning/pre-recognition period" by Clements & Battista (1990).A student who reaches 1 point is assigned to Level 1. A student who reaches 1+2=3 points is assigned to Level 2. A student who reaches 1+2+4=7 points is assigned to Level 3.A student who reaches 1+2+4+8=15 points is assigned to Level 4. A student who reaches 1+2+4+8+16=31 points is assigned to Level 5 (Usiskin, 1982).

The data collected from the VHGTLT and the GST were analyzed with the Statistical Package for social Sciences (SPSS). After performing the normality test for the research data, it was determined between which values the skewness and kurtosis coefficients changed to determine normality. When the skewness and kurtosis coefficients are in the range of +1 to -1, it can be interpreted that the data values do not deviate significantly from the normal distribution (Büyüköztürk, 2019). Since the skewness and kurtosis coefficients of the data values in the study ranged from +1 to -1, it was accepted that the data showed a normal distribution. Descriptive statistics were used to determine the level of Van Hiele geometric thinking level of Science High School students and how their geometry success was. Pearson correlation analysis was applied to examine the relation between students' Van Hiele geometric thinking levels and their geometry success. Moreover, one-factor analysis of variance (One-Way ANOVA) technique of the SPSS program was used to detect whether there was a meaningful difference between schools according to the results of VHGTLT and the GST.

Findings

The findings of the study are given below, respectively, according to the problem statements.

Findings Related to Van Hiele Geometric Thinking Levels of Science High School Students

The first sub-problem was "According to the results of the VHGTLT, what is the distribution of the Van Hiele geometric thinking levels of the Science High School students?" The findings of the question are presented in Table 3.

Table 3. Van Hiele Geometric Thinking Levels of the Study Group

Van Hiele Levels of Geometric Thinking	Frequency (f)	Percent (%)
Level 0 (No level)	7	2.9
Level 1 (Visualization)	72	29.5
Level 2 (Analysis)	10	4.1
Level 3 (Informal Inference)	119	48.8
Level 4 (Inference)	16	6.6
Level 5 (systematic Thinking)	20	8.2
Total	244	100

When Table 3 is examined, 7 students (2.9%) of the study group are Level 0 (not belonging to any level), 72 students (29.5%) are Level 1 (Visualization), 10 students (4.1%) are Level 2 (Analysis). 119 students (48.8%) were at Level 3 (Informal Inference), 16 students (6.6%) were at Level 4 (Inference), 20 students (8.2%) were at Level 5 (systematic Thinking) Van Hiele geometric thinking level is seen. It is seen that the study group students are mostly at Level 3 (Informal Inference) Van Hiele geometric thinking level. From these results, it can be said that the majority of the students in the Study group (48.8% + 6.6% + 8.2% = 63.6%) 155 students were at Level 3 (Informal Inference) Van Hiele geometric thinking level and above.

Findings Related to the Relation Between the Science High School Students' Van Hiele Geometric Thinking Level and the Geometry Success

The second sub-problem, "Is there a meaningful relationship between the Science High School students from the VHGTLT and the GST? Findings for the question" were found by calculating the Pearson correlation coefficient. The obtained results are presented in Table 4.

Table 4. The Relationbetween the Geometry Success and the Van Hiele Geometric Thinking Level

Van Hiele Geometric Thinking Levels		Geometry Success	Van Hiele Geometric Thinking Level
Geometry Success Test	r	1	0.415
	p		0.000
	N	244	244
Van Hiele Geometric Thinking Level Test	r	0.415	1
	p	0.000	
	N	244	244

When the values in Table 4 are examined, it is seen that the Pearson correlation coefficient between the GST and the VHGTLT is r = 0.415 and the level of significance is p = 0.000. In this context, it is seen that there is a positive, significant and moderate relation between Science High School students' GST and VHGTLT. The moderate relation between these two tests can be interpreted as the higher the GST, the higher the VHGTLT.

Findings Related to Van Hiele Geometric Thinking Level Test Results Among Science High Schools

The third sub-problem, "Is there a meaningful difference among the Science High Schools according to the results of the VHGTLT?" In order to find the answer to the question, first of all, the descriptive findings of the VHGTLT of the students of the three schools where the research was conducted are presented in Table 5.

Table 5. Descriptive Findings of Schools' Van Hiele Geometric Thinking Level Test

	N	Average	Standard Deflection	Standard Error	min. True Number	Max. True Number
X Science High School	60	16.68	2.75	0.35	9	21
Z Science High School	83	14	3.91	0.43	4	23
Y Science High School	101	13.78	3.80	0.37	4	21
Total	244	14.56	3.80	0.24	4	23

When the values in Table 5 are examined, it is seen that the VHGTLT averages of the students are 16.68 in X Science High School, 14 in Z Science High School and 13.78 in Y Science High School. In total, the lowest number of correct answers given to the VHGTLT is 4, and the highest number of answers is 23, and it belongs to the Z Science High School.

Findings of Levene's test applied to detect whether the variances between schools are homogeneously distributed are presented in Table 6.

Table 6. Schools' Van Hiele Geometric Thinking Level Test Levene's Test Findings

	Levene Statistics	df1	df2	Sig.(p)
Van Hiele Geometric Thinking Test Total Points	4,261	2nd	241	,015

When Table 6 is examined, the variances do not have a homogeneous distribution since p = 0.015 < 0.05 according to the findings of the Schools' VHGTLT and Levene's test.

One-Way Analysis of Variance (ANOVA) test was used to detect whether there was a difference among schools according to the results of the VHGTLT. The numerical values of the test are presented in Table 7 below.

Table 7. Interschool Van Hiele Geometric Thinking Level Test ANOVA Test Findings

Anova					
	squares total	**Df**	**squares average**	**F**	**Sig.(P)**
Between groups	357,624	2	178,812	13,662	,000
In-group	3154,191	241	13,088		
Total	3511,816	243			

When Table 7 is examined, it is seen that there is a meaningful difference among schools according to the results of the VHGTLT. (p = 0.00 < 0.05). Tamhane's T2 test, one of the post-hoc tests, was used to detect this difference. The numerical values of the test are presented in Table 8 below.

Table 8. Interschool Van Hiele Geometric Thinking Level Test Tamhane's T2 Test Findings

School (I)	**School (I)**	**Average Difference (IJ)**	**Standard Error**	**Sig.(p)**
X Science High School	Z Science High School	2,68333*	,558	,000
	Y Science High School	2,90116*	,519	,000
Z Science High School	X Science High School	-2.68333*	,558	,000
	Y Science High School	,21782	,572	,974
Y Science High School	X Science High School	-2,90116*	,519	,000
	Z Science High School	-,21782	,572	,974

When Table 8 is examined, it is seen that there is no meaningful difference between Z Science High School and Y Science High School according to the VHGTLT results. (p = 0.974 > 0.05) The average difference of 0.572 between these two schools is due to random reasons. A significant difference was found between X Science High School and Z Science High School in favor of X Science High School. (p = 0.000 < 0.05) Similarly, it is seen that there is a meaningful difference between X Science High School and Y Science High School in favor of X Science High School. (p = 0.00 < 0.05).

Findings Related to Geometry Success Test Results Among Science High Schools

The fourth sub-problem, "Is there a meaningful difference among science high schools according to the GST results? "In order to find the answer to the question, the GST descriptive findings of the students of the three schools where the research was conducted are presented in Table 9.

Table 9. Descriptive Findings of Schools' Geometry SuccessTest

	min.	**Max.**

	N	Average	Standard Deflection	Standard Error	True Number	True Number
X Science High School	60	15.45	3.67	0.47	7	21
Z Science High School	83	12.18	4.43	0.48	3	21
Y Science High School	101	11.38	4.43	0.44	2	21
Total	244	12.65	4.54	0.29	2	21

When the values in Table 9 are examined, it is seen that the GST averages of the students are 15.45 in X Science High School, 12.18 in Z Science High School, and 11.38 in Y Science High School. In total, the lowest number of true answers given to the GST is 2, and the highest number of answers is 21.

Findings of Levene's test applied to detect whether the variances between schools are homogeneously distributed are presented in Table 10.

Table 10. Schools' Geometry Success Test Levene's Test Findings

	Levene Statistics	df1	df2	Sig.(p)
Geometry Achievement test Total Points	2,270	2	241	,105

When Table 10 is examined, the variances have a homogeneous distribution since p = 0.105 > 0.05 according to the findings of the schools' GST Levene's test.

One-Way Analysis of Variance (ANOVA) test was used to determine whether there was a difference among schools according to GST results. The numerical values of the test are given in Table 11 below.

Table 11. Findings of Inter-School Geometry Success Test ANOVA Test

Anova					
	squares total	**Df**	**squares average**	**F**	**Sig.(P)**
between groups	650,002	2	325,001	17,886	,000
In-group	4379,080	241	18,170		
Total	5029,082	243			

When Table 11 is examined, it is seen that there is a meaningful difference according to the GST results among schools. (p = 0.00 < 0.05). Scheffe test, one of the post-hoc tests, was used to detect this difference. The numerical values of the test are presented in Table 12 below.

Table 12. Inter-School Geometry Success Test Scheffe Test Findings

School (I)	School (I)	Average Difference (IJ)	Standard Error	Sig.(p)

X Science High School	Z Science High School	3.26938*	,722	,000
	Y Science High School	4,06386	,694	,000
Z Science High School	X Science High School	-3,26938*	,722	,000
	Y Science High School	,79458	,631	,454
Y Science High School	X Science High School	-4,06386*	,694	,000
	Z Science High School	-,79458	,631	,454

When Table 12 is examined, it is seen that there is no meaningful difference between Z Science High School and Y Science High School according to the GST results. (p = 0.454 > 0.05) The mean difference of 0.631 between these two schools is due to random reasons. A meaningful difference was found between X Science High School and Z Science High School in favor of X Science High School. (p = 0.000 < 0.05) Similarly, it is seen that there is a meaningful difference between X Science High School and Y Science High School in favor of X Science High School (p = 0.000 < 0.05).

Findings Related to Van Hiele Geometric Thinking Levels of Science High School Students According to Geometry Success Test

The fifth sub-problem, "Does Van Hiele geometric thinking levels of science high school students show a meaningful difference compared to the GST? One-Way Analysis of Variance (ANOVA) was used to detect the answer to the question. The numerical values of the test are presented in Table 13 below.

Table 13. One-Way Analysis of Variance (ANOVA) Results of Van Hiele Levels of Geometric Thinking According to the Geometry Success Test

	Anova					
	Squares total	**Df**	**Squares average**	**F**	**Sig.(P)**	**Significant Difference**
Between groups	7871,593	5	1574,319	5,267	,000*	1-5 Level 3-5 Level
In-group	71141,653	238	298,915			
Total	79013,246	243				

When Table 13 is examined, there is a statistically meaningful difference between Van Hiele geometric thinking levels of Science High School students according to the GST (p = 0.000 < 0.05). In order to detect the reason for this meaningful difference observed between schools, pair wise comparisons between schools were made with the Scheffe test. When the pair wise comparisons between schools were examined, it was seen that the difference observed between Van Hiele geometric thinking levels according to the GST of Science High School students was statistically meaningful between students at the 1-5 and 3-5 levels. There was no statistically meaningful difference between other geometric thinking levels. Based on these results, it can be interpreted that there is a relationship between Van Hiele geometric thinking levels of Science High School students and GST.

Discussion, Conclusion and Recommendations

This research was carried out to determine the relation between Science High School students' success in geometry lesson and Van Hiele geometric thinking levels according to the results of GST and VHGTLT. In order to this aim, VHGTLT and GST were applied to 12th grade students consisting of a total of 244 students in 3 public schools in the city center of Diyarbakır, and the students' geometry thinking levels and general geometry achievement levels were examined. After the data collection process, statistical analyzes of the data were made according to the 0.05 significance level, and the results for the research problems are given below:

According to VHGTLT Results of Science High School Students, 7 (2.9%) of the students participating in the research were Level 0 (No Level), 72 (29.5%) were Level 1 (Visualization), 10 (4.1%) Level 2 (Analysis), 119 (48.8%) Level 3 (Informal Inference), 16 (6.6%) Level 4 (Inference), 20 (8.2%) Level 5 (Systematic Thinking) level. The Van Hiele geometric thinking level with the highest number of students is level 3 (visualization). According to NCTM (2000), high school students' Van Hiele geometric thinking level is expected to be Level 4 (Inference) (Knight, 2006). Similarly, Baki (2014) stated that high school students should be at the level of thinking to make inferences. From these statements, it is expected that high school students will have at least the level of Van Hiele geometric thinking at Level 3 (Informal Inference). It was determined that since the Van Hiele geometric thinking level of 155 students (63.6%) participating in the research was Level 3 and above, the Science High School students (63.6%) reached the required Van Hiele geometric level. On the other hand, the Van Hiele geometric thinking level of the students (36.4%) was lower than the required Van Hiele geometric thinking level. For this reason, it is important for the success of these students to consider the Van Hiele geometric thinking levels of these students when planning geometry lessons.

When the relevant studies in the literature are examined, Hurma's (2011) study with high school 9th grade students, Altun's (2018) study with 11th grade high school students and Usiskin's (1982) study with high school students in the USA. The students' Van Hiele geometric thinking levels in these studies were lower than the level they thought they should reach. The findings obtained in this study are that the Van Hiele geometric thinking levels of Science High School students are better than the findings obtained in the mentioned studies.

The correlation coefficient between the GST prepared by the researcher according to the in the high school curriculum and the VHGTLT was $r = 0.415$ and the level of significance was $p = 0.00$. The fact that $p < 0.05$ and the correlation coefficient is considerably higher than 0 indicates that there is a positive and significant relationship between the GST and the VHGTLT. In this context, it was concluded that the relation between the two tests was moderate and as the VHGTLT score increased, the GST scores would also increase.

When the schools were examined in detail, the VHGTLT averages were found to be 16.68 in X Science High School, 13.78 in Y Science High School, and 14 in Z Science High School. When the findings of the ANOVA test in terms of the VHGTLT were examined among the schools, it was seen that there was no significant difference between Z Science High School and Y Science High School, and that there was significant difference between X Science High School, which received the highest score in the high school entrance exam in the province, and Z Science High School and Y Science High School. It is seen that there is a significant difference in favor of X Science High School.

When the schools were examined in detail, the GST averages were found to be 15.45 in X Science High School, 11.38 in Y Science High School, and 12.18 in Z Science High School. When the ANOVA test findings are examined in terms of GST between schools, it is seen that there is no meaningful difference between Z Science High School and Y Science High School, and that there was significant difference between X Science High School, which received the highest score in the high school entrance exam in the province, and Z Science High School and Y Science High School. It is seen that there is a significant difference in favor of X Science High School.

It is seen that there is a statistically meaningful difference between the Van Hiele geometric thinking levels of the Science High School students according to the GST. When the pair wise comparisons between schools were examined, it was found that the difference observed between Van Hiele geometric thinking levels according to the GST of Science High School students was statistically significant between student levels at 1-5 and 3-5.

In our study, we found that the Science High School students had different levels of Van Hiele geometric thinking and the relationship between the Van Hiele geometric thinking levels test and the GST was moderate. We see that there is a meaningful difference in favor of the Science High School, which receives students with the highest score in the high school entrance examination, and the difference between Van Hiele geometric thinking levels is statistically meaningful between the student levels at the 1-5 and 3-5 levels.

Based on all these results, the necessity of determining the Van Hiele geometric thinking levels of the students before teaching geometry, the necessity of progressing along the order of geometric thinking levels while teaching geometry (Usiskin, 1982: 3) and the necessity of processing geometry teaching according to the students' Van Hiele geometric thinking levels have emerged.

Suggestions for some future work are given below.

This study is about all geometry subjects in high school curriculum. In the high school and middle school geometry curriculum, different studies can be done on different subjects, more specific.

Since different variables (algebraic thinking, hypothetical thinking, etc.) can be effective on students' Van Hiele geometric thinking levels, studies can be conducted with variables other than geometry success.

Qualitative studies can be conducted to deeply examine the relation between Van Hiele geometric thinking levels and geometry success or different learning models.

Significant differences were found between schools according to Van Hiele geometric thinking level and GST in the schools where the research was conducted. The reason for these differences can be investigated.

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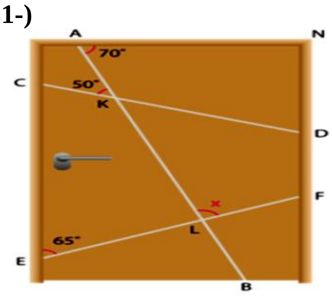
Appendix

SampleQuestions of GST

S-1-)

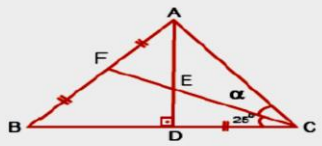


Bir mobilyacı, siparişini aldığı kapı ile ilgili ölçüleri yukarıdaki gibi olan tasarımı yapmıştır.
Şekilde $m(\widehat{AKC}) = 50°$, $m(\widehat{CEF}) = 65°$, $m(\widehat{NAK}) = 70°$ ve $m(\widehat{ALF}) = x$ olarak verilmiştir

Buna göre x kaç derecedir?

A) 75
B) 80
C) 85
D) 90
E) 95

S—2-)

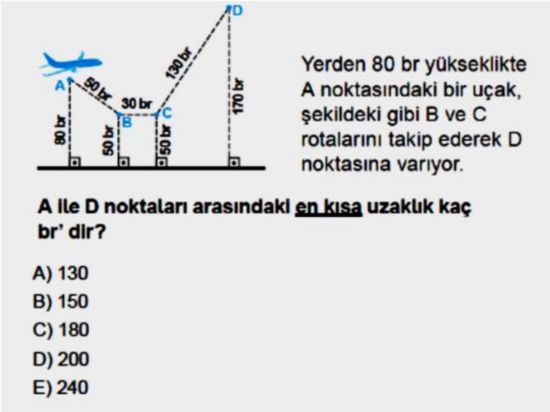


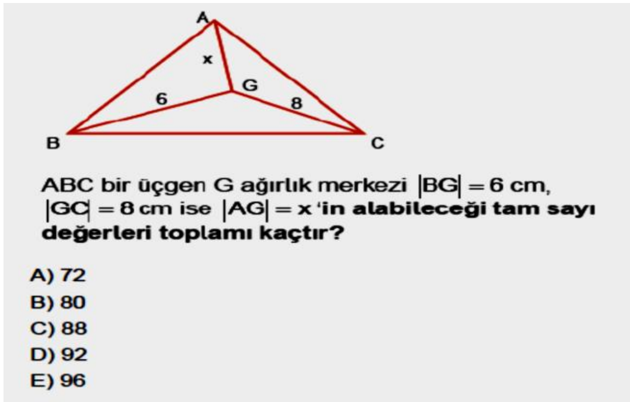
ABC bir üçgen, $[AD] \perp [BC]$, $|AF| = |FB| = |DC|$ $|AC| = |FC|$, $m(\widehat{BCF}) = 25°$ olduğuna göre, $m(\widehat{ACF}) = \alpha$ kaç derecedir?

A) 20
B) 30
C) 35
D) 40
E) 45

S-3-)

S-4-)





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