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How Procedural and Conceptual Knowledge in Math Course Drive Analytical Thinking Development

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ABSTRACT

The aim of this study is to examine the effects of instructional processes in mathematics courses, grounded in conceptual and procedural knowledge, on the analytical thinking skills of primary school students. To achieve this objective, a quasi-experimental design was employed. The study group consisted of 48 fourth-grade students enrolled in a primary school during the 2023–2024 academic year. The students' analytical thinking skills were measured using a 15-item multiple-choice academic achievement test developed by the researcher. The test assessed core components of analytical thinking, including comparison, part-whole relationships, cause-effect relationships, classification, and sequencing. Prior to data analysis, key statistical assumptions—normality, homogeneity of variances, and equality of variance-covariance matrices—were tested. An independent samples t-test was conducted to assess whether the groups were equivalent in terms of their pre-test scores. To examine the effects of instructional type and time, a mixed-design ANOVA was utilized. Paired samples t-tests were used to analyze within-group differences, and an additional independent samples t-test was conducted to evaluate between-group differences in post-test scores. The findings indicated that instructional processes based on conceptual knowledge were more effective in improving students' analytical thinking skills than those based on procedural knowledge.

Keywords: Conceptual Knowledge, procedural knowledge, analytical thinking, mathematics education.

Introduction

In the past, mathematics education primarily focused on equipping learners with the ability to perform mathematical operations. However, contemporary approaches emphasize understanding why specific operations are performed and how the resulting outcomes can be applied across various fields. This shift is driven by the increasing complexity and data-intensive nature of modern problems. Therefore, identifying which data are necessary, why they are needed, and how much they contribute to solving a problem plays a crucial role in reaching effective solutions (Ye, et al., 2023). In other words, instead of merely performing operations, the essence of mathematics education has become knowing which operations to perform, why, and when, through analytical thinking. According to Sebetci and Aksu (2014), mathematics fosters analytical thinking through the abstract concepts it encompasses. Analytical thinking involves several steps, such as identifying the essential components of a problem, analyzing the relationships between these components, and constructing systematic solutions (Leron & Hazzan, 2009). Swartz and Parks (1994) define analytical thinking as a cognitive process that includes skills such as classification, sequencing, part-whole reasoning, cause-effect analysis, and comparison. These skills enable individuals to accurately identify the nature of a problem and what is required to solve it. Thus, this process demands a level of cognitive engagement far beyond simply performing prescribed mathematical operations. Among various thinking skills, analytical thinking holds particular importance in domains involving numerical and logical reasoning. It allows students to break down complex information, understand its components, and systematically evaluate relationships between them. This makes analytical thinking an indispensable component of mathematical learning. Mathematics, by its nature, requires abstract thinking, establishing cause-and-effect relationships, and generating systematic solutions. Therefore, a reciprocal relationship exists between mathematics education and the development of analytical thinking. Analytical thinking fosters the emergence of solutions that may seem initially unrelated but ultimately complement one another. This type of thinking refers to the ability to identify the necessary components of a problem, analyze their interconnections, and construct solutions in a systematic manner (Olkun & Toluk, 2003).

In this context, the aim of mathematical problem-solving is not merely to arrive at the correct answer but to understand the process and justify it logically. Analytical thinking helps students develop more structured and meaningful approaches to problem-solving, thus facilitating deeper comprehension of mathematical concepts. However, for this mode of thinking to be effectively cultivated, students must understand not only the procedural steps but also the conceptual foundations underlying these steps. At this point, two key types of knowledge frequently discussed in mathematics education—procedural and conceptual knowledge—become relevant. Procedural knowledge consists of the rules, formulas, and symbols necessary to perform mathematical operations. This type of knowledge is typically associated with rote memorization and the step-by-step execution of procedures, often without logical justification (Olkun & Toluk, 2003). In essence, it is about knowing how, rather than why, something is done. It includes recognizing symbols correctly, applying formulas and rules, following specific sequences, and adhering to algorithmic steps (Hiebert & Lefevre, 2013). Individuals with procedural knowledge often solve mathematical problems using memorized techniques. This type of knowledge is particularly effective in solving routine and standard problems. Rittle-Johnson and Schneider (2013) assert that procedural knowledge helps automate problem-solving processes, as frequently used procedures eventually require less cognitive effort.

Conceptual knowledge, on the other hand, refers to the understanding of mathematical concepts and their components, the ability to explain these concepts using symbolic representations, and the capacity to grasp the methods of operations while establishing meaningful connections between symbols, procedures, and concepts (Comrey & Lee, 1992). Conceptual understanding is achieved through the balanced integration of both conceptual and procedural knowledge, playing a crucial role in equipping students with advanced problem-solving skills (Lloyd, et al.,2010). Skemp (2012) defines conceptual knowledge as “knowing what to do, how to do it, and why it is done that way”. Based on this perspective, the present study seeks to address the question: Does mathematics instruction based on the conceptual knowledge model significantly differ from that based on the procedural knowledge model in terms of its effect on students' analytical thinking skills? Accordingly, the research is guided by the following hypotheses:

1.The change in scores between the pre-test and post-test significantly differs between students who receive mathematics instruction based on the conceptual model and those who receive instruction based on the procedural model.

2.There is no significant difference in the pre-test scores between students taught using the conceptual model and those taught using the procedural model.

3.There is a significant difference between the pre-test and post-test scores of students who receive mathematics instruction based on the conceptual model.

4.There is a significant difference between the pre-test and post-test scores of students who receive mathematics instruction based on the procedural model.

Methods and Materials

To evaluate the effect of instructional processes supported by procedural and conceptual knowledge on students' analytical thinking skills, two groups participating in different instructional models were compared (between-group comparison). Furthermore, by including repeated measures from the same participants, the study was designed using a pre-test–post-test control group experimental design. Since random assignments were not applied and intact classrooms were designated as experimental and control groups, the study employed a quasi-experimental design (Borji et al., 2021). The study group consisted of 48 fourth-grade students enrolled in two different classes within the same primary school located in a metropolitan city. The experimental group comprised 24 students (11 female and 13 male), while the control group also consisted of 24 students (14 female and 10 male).

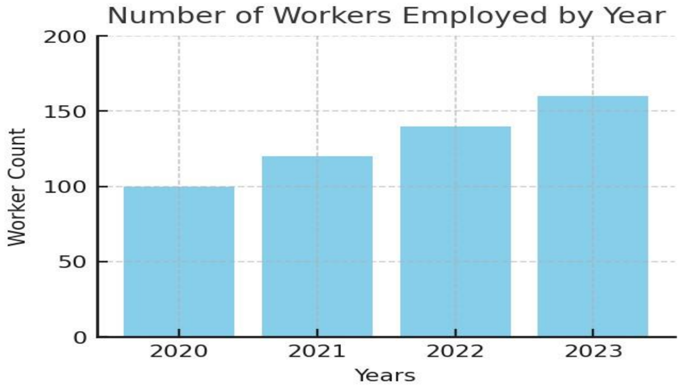
Data Collection Instrument

As part of the research process, an academic achievement test was developed. To construct this test, 20 items were written based on analytical thinking and aligned with the intended learning outcomes of the fourth-grade math course. These items were revised based on experts' (one of them had PhD. degree in mathematics and the other one is a professor in the field of instruction) opinions regarding clarity, alignment with visuals, and age appropriateness. The revised version of the test was then administered to 225 fifth-grade students for pilot testing. Considering that comparison, part-whole relationships, cause-effect reasoning, classification, and sequencing together represent a holistic analytical thinking ability, item selection was guided by the total discrimination indices (rjx) of the scale. As a result of the item analysis, the discrimination indices of the initial 20-item test ranged from –.15 to .53. From each sub-dimension, the three items with the highest discrimination indices were selected and reanalyzed. The final application form consisted of 15 items with discrimination indices ranging from .31 to .79. The difficulty levels (pj) of these items ranged from .35 to .85. The reliability of the 15-item form was calculated using the KR-20 coefficient, which was found to be .75. Given that all items had discrimination indices above .30 and the reliability exceeded .70, the final version of the application form was accepted. Items 1–3 represent the comparison dimension, items 4–6 assess part-whole relationships, items 7–9 focus on cause-effect reasoning, items 10–12 address classification, and items 13–15 cover sequencing.

One of the items on sequencing skills is given below:

The number of workers employed in a factory by year is shown in the chart on the right.
If the increase in the number of workers continues as shown in the chart, in which year will the number of workers in the factory exceed 240?

A)2026
B)2027
C)2028
D) 2029



Procedure

The finalized achievement test was administered as a pre-test to all fourth-grade student groups. Lessons were then designed separately based on either the procedural or conceptual knowledge model, in alignment with the national curriculum outcomes. While the experimental group received instruction based on the conceptual model, the control group was taught using the procedural model. Both groups received 40 hours of instruction in total. The learning outcomes and respective instructional hours (as indicated national curriculum) were as follows: explaining the relationship between units of time (4 hours), solving problems involving units of time (6 hours), making interpretations and predictions based on graphs (10 hours), creating bar graphs and using various representations to present data (10 hours), and solving real-life problems using bar graphs, tables, and other visual data representations (10 hours). In the procedural model-based instruction, the emphasis was placed on the outcomes of operations and error correction. Students were presented with traceable algorithms, and operations were carried out step-by-step with guided practice and examples. Students were encouraged to independently complete similar problems. The correct sequence of operations, special rules, and shortcuts were highlighted. For students who made mistakes, the step where the process was broken was explained and the correct steps were repeated together with the student. Conversely, in conceptual model-based instruction, lessons began with activities that activated students' prior knowledge and presented real-life examples of the concept. Concept maps, brainstorming, visuals, and concrete materials were used to create connections. The same concept was presented in visual, verbal, numerical, and symbolic forms. Students were asked exploratory questions such as "Why?", "How?", and "What difference does it make?". Open-ended questions were used to explore the boundaries and applications of the concepts. Additionally, the learning outcomes were supported by real-life problem scenarios or modeling activities.

Ethical Process

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards This study was conducted with the approval of the Human Research Ethics Committee of Aksaray University in Turkey, under the decision dated October 23, 2023 (Reference No: 2023/06-39). Due to the participants being primary school students, their parents were informed about the research process and written consent was obtained. Additionally, the necessary permissions from the Provincial Directorate of National Education were secured to conduct the implementation.

Data Analyses

Before initiating the data analysis process, assumptions for parametric testing were evaluated using the data collected after the implementation. The normality of the dataset was assessed using skewness-kurtosis coefficients and the Shapiro-Wilk test. Homogeneity of variances was tested with Levene's test, and equality of variance-covariance matrices was examined using Box's M test. First, an independent samples t-test was conducted to determine whether the groups had equivalent pre-test scores. Then, a mixed-design ANOVA was used to analyze the effects of time (pre-test–post-test) and instructional model (procedural–conceptual) on analytical thinking skills. To explore within-group differences between pre-test and post-test scores, paired samples t-tests were performed separately for each group. Finally, an independent samples t-test was conducted to evaluate the differences in post-test scores between the groups. The significance level for all analyses was set at .05.

Findings

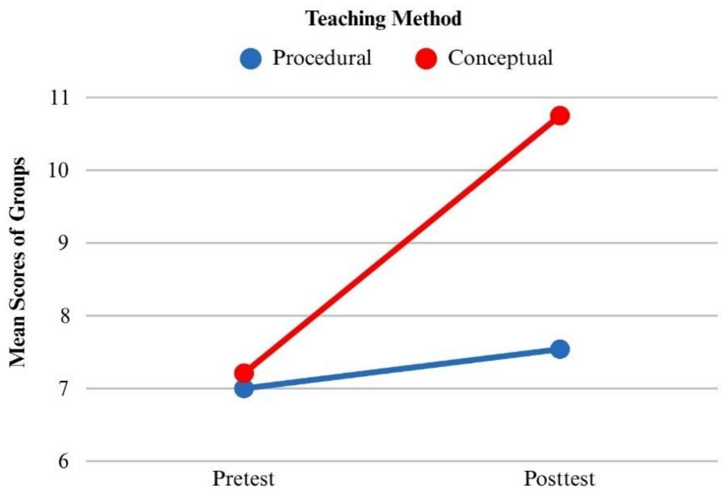
Descriptive statistics regarding the participants' pre-test and post-test scores are presented in Table 1. To determine whether the groups were equivalent prior to instruction, an independent samples t-test was conducted on the pre-test scores. Before the analysis, it was observed that the skewness and kurtosis coefficients fell within the acceptable range of –1.5 to +1.5 and the Shapiro-Wilk test was non-significant ($p > .05$), indicating that the assumption of normality was met. Additionally, Levene's test confirmed the homogeneity of variances ($p > .05$). No significant difference was found between the pre-test scores of the groups receiving instruction based on procedural knowledge ($n = 24$, M = 7.00, SD = 1.72) and those receiving instruction based on conceptual knowledge ($n = 24$, M = 7.21, SD = 2.08), $t(46) = –0.38$, $p = .707$. Accordingly, the groups were considered equivalent in terms of analytical thinking skills at the outset.

Table 1. Descriptive Statistics of Pre-test and Post-test Scores by Group

Types of Models	Test Type	Mean	Standart Deviaton
Procedural (n = 24)	Pre-test	7.00	1.72
	Post-test	7.54	1.86
Conceptual (n = 24)	Pre-test	7.21	2.08
	Post-test	10.75	1.96
Total (n = 48)	Pre-test	7.10	1.89
	Post-test	9.15	2.49

To examine the effect of instructional processes supported by procedural and conceptual knowledge on analytical thinking skills, a mixed-design ANOVA was conducted. Prior to the analysis, the assumptions were tested. The skewness and kurtosis coefficients of both groups' pre-test and post-test scores were found to be within the acceptable range of –1.5 to +1.5, and the Shapiro-Wilk test was not significant ($p > .05$), indicating approximately normal distributions. Levene's test confirmed the homogeneity of variances ($p > .05$), and Box's M test verified the equality of variance-covariance matrices ($p > .05$). Since all assumptions were met, the mixed-design ANOVA was performed. First, the interaction between instructional process and time was examined and found to be statistically significant ($F(1, 46) = 29.59$, $p < .001$, $\eta^2_p = .391$). This result indicates that the change in scores from pre-test to post-test differed significantly between the groups. As illustrated in Figure 1, the post-test improvement among students who received instruction based on conceptual knowledge was greater than that of those who received instruction based on procedural knowledge.

Figure 1. Changes in Pre-test and Post-test Scores by Instructional Groups



After identifying the interaction effect, simple main effects were analyzed to examine the differences between the groups' pre-test and post-test scores. Paired samples t-tests were conducted for each group. For the procedural group, the difference between the pre-test (M = 7.00, SD = 1.72) and post-test (M = 7.54, SD = 1.86) scores was not statistically significant ($t(23) = -1.67$, $p = .108$). However, for the conceptual group, the difference between the pre-test (M = 7.21, SD = 2.08) and post-test (M = 10.75, SD = 1.96) scores was statistically significant in favor of the post-test ($t(23) = -7.94$, $p < .001$). Finally, the post-test scores of the two groups were compared using an independent-samples t-test. The results showed that the scores of the conceptual group (M = 10.75, SD = 1.96) were significantly higher than those of the procedural group (M = 7.54, SD = 1.86), ($t(46) = -5.81$, $p < .001$).

Results and Discussion

The analysis addressing the hypothesis "There is no significant difference between the pre-test scores of students taught with the conceptual and procedural knowledge models" confirmed that the groups were equivalent in terms of analytical thinking skills before instruction. These findings support the hypothesis, indicating that no significant difference existed between the groups' pre-test scores (Figure 1). This result aligns with the findings of Borji et al., (2021), who also emphasized cognitive equivalence between groups before instruction. Similarly, Baroody and Johnson (2007) stated that initial equivalence enhances the validity of research findings. Newton, et al., (2010) also highlighted the importance of comparable baseline levels to evaluate the impact of procedural and conceptual instruction. These findings are consistent with the literature and confirm the equivalence of groups prior to the intervention. In experimental and quasi-experimental research, initial equivalence is essential to validly assess the effects of instructional practices (Tabachnick & Fidell, 2013). Accordingly, the pre-test results ensured group homogeneity and allowed for an objective evaluation of the instructional impact on analytical thinking skills.

Analyses addressing the hypothesis “The change in pre-test and post-test scores significantly differs between students taught with conceptual and procedural knowledge models” revealed a statistically significant difference in the development of analytical thinking skills between the two instructional approaches. Notably, students in the conceptual knowledge group showed a more pronounced improvement in their post-test scores, supporting this hypothesis. Hussein and Csíkos (2023) stated that instruction based on conceptual knowledge enhances mathematical achievement, reduces anxiety, and fosters positive attitudes. Similarly, Samad et al., (2022) identified a significant correlation between conceptual understanding and analytical thinking skills. In a microlearning-based study by Alptekin (2025), students demonstrated high levels of achievement in understanding algebraic concepts. Anderson et al., (1999) emphasized that conceptual instruction promotes deeper understanding and critical thinking. The mixed-design ANOVA results revealed a significant interaction effect ($F(1, 46) = 29.59$, $p < .001$, $\eta^2_p = .391$), further supporting the research hypothesis by demonstrating that the instructional process plays a critical role in the variation of student achievement over time. Brown and Coles (2010) also stressed the importance of instructional strategies that consider individual differences for effective learning. In this context, the findings suggest that different instructional approaches produce varying levels of academic development among students.

The hypotheses “There is a significant difference between pre-test and post-test scores among students taught with the conceptual knowledge model” and “There is a significant difference between pre-test and post-test scores among students taught with the procedural knowledge model” were tested. The analyses indicated a significant and substantial increase in the post-test scores of the conceptual instruction group, whereas the procedural instruction group did not exhibit a similar improvement. These results support both hypotheses and confirm the effectiveness of conceptual knowledge in enhancing analytical thinking skills. Similar findings have been reported in the literature. Hiebert et al., (2002) emphasized the pivotal role of conceptual knowledge in problem-solving and critical thinking. Similarly, Booth et al., (2013) highlighted that conceptual instruction helps students understand the rationale behind procedures and fosters higher-order cognitive skills. In mathematics education, conceptual knowledge facilitates understanding of mathematical structures, while procedural knowledge focuses on the application of rules and algorithms (Star, 2005). Although a balanced integration of both types of knowledge is recommended, the present study found that conceptual knowledge is more effective in developing analytical thinking. Rittle-Johnson and Lefevre (2013) noted that analytical thinking supports understanding of relationships between mathematical concepts and promotes flexibility in problem-solving. Conversely, procedural instruction does not appear to foster this skill, and its sole focus on procedures may hinder students' ability to form conceptual connections (Hiebert & Carpenter, 2002). In a study conducted by Uz (2022) with middle school students, it was observed that students predominantly used procedural strategies in problem-solving, while their mathematical modeling competencies remained low. This finding suggests that lacking conceptual grounding in procedural knowledge can impede knowledge transfer to new situations. Therefore, it is recommended that instructional approaches grounded in conceptual knowledge be prioritized to develop students' analytical thinking skills. Activities should be designed to enable students to understand mathematical structures and apply their knowledge to novel problems. Furthermore, instructional programs that integrate both conceptual and procedural knowledge in a balanced manner can enhance students' progress in both fundamental skills and higher-order thinking (Rittle-Johnson& Lefevre, 2013). Yarka (2024)’s study revealed that modeling activities significantly improved students’ levels of conceptual and procedural knowledge, particularly in operations with fractions.

In both everyday life and learning processes, many challenges or tasks cannot be addressed solely through possessing knowledge; rather, they require meaningful interpretation and effective application of that knowledge. This highlights the importance of critical thinking, analytical reasoning, and creative problem-solving skills. Making knowledge meaningful involves not just memorizing information obtained from one's environment, but also analyzing, connecting, and evaluating it within context. For instance, a student does not merely need to know a mathematical formula; they must also understand when, where, and how to use it. In this process, individuals activate prior knowledge, relate new information to existing knowledge, establish conceptual connections, and organize information into mental schemas. This cognitive organization transforms knowledge into long-term learning and supports effective problem-solving. Meaningful knowledge becomes a tool for generating solutions. The student or individual utilizes their knowledge to propose alternative solutions, compare these alternatives, and choose the most appropriate one. Once the solution is determined, it is implemented. However, the problem-solving process does not end here. The individual evaluates whether the solution is effective and sufficient. If necessary, they experiment with alternative approaches. This iterative process fosters the development of self-evaluation and self-regulation skills.

Result

Based on all these findings, it can be concluded that instructional processes grounded in conceptual knowledge are more effective in enhancing students' analytical thinking skills compared to those based on procedural knowledge. Conceptual instruction enables students to comprehend the relationships among mathematical concepts, resulting in significant improvements in their problem-solving and critical thinking abilities. In contrast, instructional approaches focused primarily on procedural knowledge appear to have a limited impact on the development of analytical thinking, often leaving students confined to procedural understanding alone. These results highlight the need to prioritize conceptual understanding in mathematics education and underscore the importance of designing curricula that integrate conceptual and procedural knowledge in a balanced and complementary manner.

Limitations and Recommendation

This study has several limitations that should be acknowledged. First, the sample was limited to 48 fourth-grade students from a single primary school, which restricts the generalizability of the findings. Future research should include a larger and more diverse sample across different educational settings to enhance external validity. Second, the study employed a quasi-experimental design without random assignment, which may limit the control over confounding variables. Employing randomized controlled trials in subsequent studies could strengthen causal inferences. Third, the measurement of analytical thinking was confined to a multiple-choice test developed by the researcher. While the test was aligned with the core components of analytical thinking, the inclusion of open-ended or performance-based assessments could provide a more comprehensive understanding of students' cognitive processes.

Based on these limitations, several recommendations are proposed. Researchers should consider implementing longitudinal studies to examine the long-term effects of conceptual and procedural instruction on analytical thinking. It is also recommended that future studies explore the impact of blended instructional approaches that integrate both knowledge types within authentic learning environments. Finally, teacher training programs should emphasize the importance of conceptual knowledge and equip educators with strategies to foster analytical thinking through meaningful classroom practices.

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